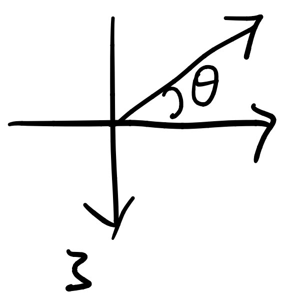
1. a) 22/11/24

$\vec{v} = 6$



Let the river flow in the $-y$-axis direction at 3mph and the $+x$-axis as the direction straight across the river.

$$\vec{v} \sin\theta = 3$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$

$\therefore \frac{\pi}{6}$ from the positive x-axis.

b) The rower is not able to go straight across the river as the vertical velocity component of 3mph can never cancel out the river velocity at 6mph.

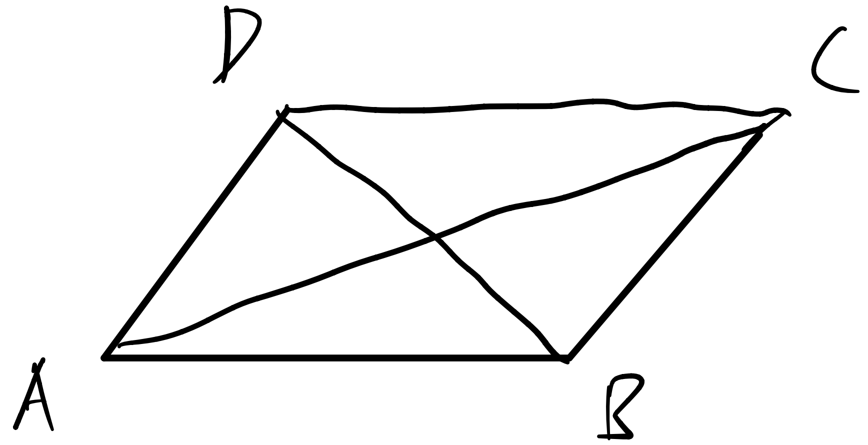
2. $\langle 2, 3 \rangle$

$$= \frac{1}{|v|} \langle 2, 3 \rangle$$

$$= \frac{1}{\sqrt{2^2+3^2}} \langle 2, 3 \rangle$$

$$= \frac{1}{\sqrt{13}} \langle 2, 3 \rangle$$

3. 19/6/25



Let M_1 be the midpoint of AC and M_2 be the midpoint of BD.

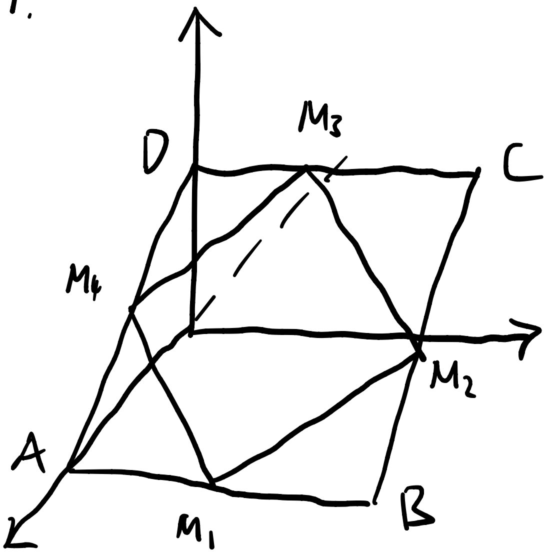
We want to show that $\overrightarrow{AM_1} = \overrightarrow{AM_2}$

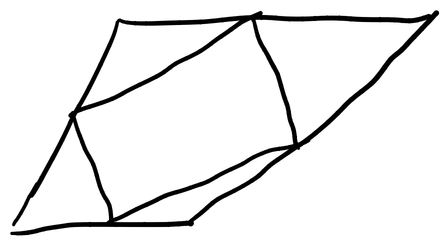
$$\overrightarrow{AM_1} = \frac{1}{2}\overrightarrow{AC} \qquad \overrightarrow{AB} = \overrightarrow{DC} = \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{DC}$$

$$\begin{aligned}
\overrightarrow{AM_2} &= \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BD} \\
&= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{DC} + \frac{1}{2}\overrightarrow{BD} \\
&= \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{BD} + \overrightarrow{DC}\right) \\
&= \frac{1}{2}\overrightarrow{AC}
\end{aligned}$$

$\therefore \overrightarrow{AM_1} = \overrightarrow{AM_2}$. $\square$

4. 20/6/25





Let M_1 be the midpoint of AB, and M_2, M_3, M_4 be the midpoints of BC, CD, and DA respectively.

If $M_1M_2M_3M_4$ is a parallelogram, $\overrightarrow{M_1M_4} = \overrightarrow{M_2M_3}$.

$\overrightarrow{M_2M_4} = \overrightarrow{M_2M_1} + \overrightarrow{M_1M_4}$

$\overrightarrow{M_2M_4} = \overrightarrow{M_2M_3} + \overrightarrow{M_3M_4}$

$\Rightarrow \overrightarrow{M_2M_1} + \overrightarrow{M_1M_4} = \overrightarrow{M_2M_3} + \overrightarrow{M_3M_4}$

$\Rightarrow (\overrightarrow{M_2B} + \overrightarrow{BM_1}) + \overrightarrow{M_1M_4} = \overrightarrow{M_2M_3} + (\overrightarrow{M_3D} + \overrightarrow{DM_4})$

Because $\overrightarrow{M_2B} = \overrightarrow{CM_2}$, $\overrightarrow{BM_1} = \overrightarrow{M_1A}$, $\overrightarrow{M_3D} = \overrightarrow{CM_3}$, $\overrightarrow{DM_4} = \overrightarrow{M_4A}$

$\Rightarrow \overrightarrow{CM_2} + \overrightarrow{M_1A} + \overrightarrow{M_1M_4} = \overrightarrow{M_2M_3} + \overrightarrow{CM_3} + \overrightarrow{M_4A}$

$\overrightarrow{M_1A} - \overrightarrow{M_4A} + \overrightarrow{M_1M_4} = -\overrightarrow{CM_2} + \overrightarrow{CM_3} + \overrightarrow{M_2M_3}$

$\overrightarrow{M_1A} + \overrightarrow{AM_4} + \overrightarrow{M_1M_4} = \overrightarrow{M_2C} + \overrightarrow{CM_3} + \overrightarrow{M_2M_3}$

$2\overrightarrow{M_1M_4} = 2\overrightarrow{M_2M_3}$

$\therefore \overrightarrow{M_1M_4} = \overrightarrow{M_2M_3}$. $\square$